Mastering minor piece endgames
Part 2
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KEY TO SYMBOLS

= Equality or equal chances
± White has a slight advantage
† Black has a slight advantage
± White is better
† Black is better
+- White has a decisive advantage
-+ Black has a decisive advantage
∞ unclear
∞ with compensation
⇔ with counterplay
↑ with initiative
→ with an attack
△ with the idea
□ only move

N novelty
! a good move
!! an excellent move
? a weak move
?? a blunder
!? an interesting move
?! a dubious move
+ check
# mate
INTRODUCTION

The third book of the series “The Modern Endgame Manual” is about the fight between the bishop and the knight, and also covers purely knight endings, and is the 2nd book on ‘minor pieces endgames’: in the first one we focus only on bishop endgames.

The knight is a short range piece, but a strong one. It is the only piece that can “fork” several pieces at the same time, which can lead to a quick material win. On the other hand, it takes 3 full moves to get the knight from the square e4 to e5 — and a full 4 moves to get the knight from e4 to c6!

Knight endgames can usually be evaluated in the same way as pawn endgames.

The most difficult pawns to handle for the knight are clearly the a- and h-pawns, because they limit the knight’s activity — it can’t step off the edge of the board!

However, when standing in the center of the board, a knight can be a very powerful piece indeed: able to play on both flanks at the same time. Unfortunately, in a battle of ‘stopping a running pawn’, for example, the knight is a very slow and cumbersome beast.

In the section of the book where we look at Bishop vs Knight we do not have so many theoretical positions to consider and so it comes down more to practical knowledge. But still, a few basic rules clearly apply:

In open positions the bishop is a much stronger piece. For example, a bishop can stop the a-pawn and at same time support his h-pawn, whereas the knight can be useful only on one wing at a time. But in closed positions exactly the opposite occurs, and the domination by the knight is often clear to see.

In pure knight endings, passed pawns usually play the crucial role — especially the a- and h- pawns. A material advantage also usually gives the stronger side very good chances to bring home the full point.
The material inside this book is very useful for players at every level, from the complete beginner up to GM standard! After a deep study of this book you will be able to call yourself quite an expert, something you will hopefully prove in your future games!
In this series of nine endgame books, FIDE Senior Trainer Adrian Mikhalchishin, FIDE Senior Trainer Efstratios Grivas and IGM Csaba Balogh combine their experience as trainers and as practical players to create something very special.

The authors aim for very understandable explanations of every endgame position in each book.

The specification:
- 1st book — Queen and pawn endgames.
- 2–3 — Minor piece endgames (bishop and knight endgames).
- 4–5 — These will feature the fight between different material constellations.
- 6–8 — These books are going to focus on the most common endgames, which are of course rook endings.
- 9th — This book will focus on exchanges and simplifications.

The main concept of each book is to provide theoretical knowledge which can be used in practical games. It means the focus of the books will be on positions which are the most likely to occur - and the practical playing of them.

That’s why you will firstly meet the theoretical part, and secondly the practical examples of how games actually continued in a particular endgame.

Yes, you’re right, you won’t find too many very complicated studies, stunning manouevres or rarely-appearing positions — and there is a simple reason why not!

How often do we see positions, for example two knights vs pawn where one knight is blocking the pawn and the other one tries to get the king to the corner before releasing the second knight for the mate? Or constellations with crazy material on the board? This might happen in one game out of 100! You could spend hundreds of hours working on something that might bring you “only” a single point more out of 100 games!
Our approach is quite different: let’s make more points in the other 99 games! And who knows, we might also be lucky in the remaining one, but actually, statistically, it would almost not matter.

“The Modern Endgame Manual” will make an expert out of you in most of the endgames which are going to appear in your long career as a chess player!
CHAPTER 1

KNIGHT VS PAWN

The knight is a short-ranged piece, but even short distances can take it quite some time. For instance, from e4 to e5, it requires 3 moves, or from e4 to c6, it takes 4 moves to arrive there. On the other hand, sometimes it is able to stop promoting pawns by coming from the most distant part of the board, often winning a tempo with a check along the way.

Knight endgames often have to be evaluated in the same way as pawn endgames, as passed pawns play perhaps the most important role here as well.

We must always take some time to consider the transpositions into pawn endgames and we should also pay attention to how we spend our tempi with the pawns, because zugzwangs are also common subjects and it is relevant to keep in mind that knights cannot lose a tempo. There is no triangulation with the knight, because the knight can only arrive to the same square in an even number of moves.

1.1. KNIGHT VS 1 PAWN

In this chapter we will examine how the knight can fight against a passed pawn. Probably the most frequently seen practical example is when the h-pawn has reached the 6th rank and the knight stands in front if it on h7. In this case the knight holds the pawn even without the help of the king.

![Theoretical position](image)

A single knight cannot create a mate by itself, however if the opponent has
an h-pawn and his king is squeezed in front of it to the corner, the side with the knight might win. Here is the classic example.

1. \text{f1}!

and Black is forced to self-mate himself with

1... \text{h2} 2. \text{g3#}

White cannot checkmate as he cannot give the right to move to Black—in other words, he cannot lose a tempo.

1... \text{h1} 2. \text{f2} \text{h2}

The last step is to get the knight to f1! It cannot be reached in the most direct way with \text{g3} because of stalemate.

3. \text{c3}

Another route is 3. \text{d4} \text{h1} 4. \text{f5} \text{h2} 5. \text{e3} \text{h1} 6. \text{f1+-}

3... \text{h1} 4. \text{e4}

It is very important, when the king is on h1, that White finds a square for his knight from where it prevents the self-stalemate idea with h2 by threatening \text{g3} mate.
4...\textit{h}2 5.\textit{d}2 \textit{h}1 6.\textit{f}1! \textit{h}2 7.\textit{g}3#  

\textbf{3}  

Theoretical position

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & \textbf{a} \\
\hline
7 & 6 & 5 & 4 & 3 & 2 & 1 & \text{b} & \\
\hline
6 & 5 & 4 & 3 & 2 & 1 & \text{c} & \text{d} & \\
\hline
5 & 4 & 3 & 2 & 1 & \text{e} & \text{f} & \text{g} & \\
\hline
4 & 3 & 2 & 1 & \text{f} & \text{g} & \text{h} & \\
\hline
3 & 2 & 1 & \text{g} & \text{h} & \\
\hline
2 & 1 & \text{h} & \\
\hline
8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & \textbf{a} \\
\hline
\end{tabular}
\end{center}

A slightly more complex, but ultimately achievable, mate:

1.\textit{g}4! \textit{h}3 2.\textit{f}2+!

Only by this subtle maneuver can White reach the ideal setup of the pieces.

2...\textit{h}2 3.\textit{e}4 \textit{h}1 4.\textit{f}2!

And the rest we already know.

4...\textit{h}2

4...h2 5.\textit{g}3#

\textbf{4}  

Theoretical position

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & \textbf{a} \\
\hline
7 & 6 & 5 & 4 & 3 & 2 & 1 & \text{b} & \\
\hline
6 & 5 & 4 & 3 & 2 & 1 & \text{c} & \text{d} & \\
\hline
5 & 4 & 3 & 2 & 1 & \text{e} & \text{f} & \text{g} & \\
\hline
4 & 3 & 2 & 1 & \text{f} & \text{g} & \text{h} & \\
\hline
3 & 2 & 1 & \text{g} & \text{h} & \\
\hline
2 & 1 & \text{h} & \\
\hline
8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & \textbf{a} \\
\hline
\end{tabular}
\end{center}

In the rest of this chapter we are going to study how the knight can fight against a passed pawn. We should never forget that the knight is a very tricky piece, one which can always give some unexpected checks. The pawn cannot be stopped, but the draw can be held by

1.\textit{d}4!

But not 1.\textit{g}5? because of \textit{e}3! taking control over the f3 square and the pawn promotes.

1...\textit{e}1= 2.\textit{f}3+ \textit{e}2 3.\textit{x}e1=
Against the e-f-g-pawns, the knight holds the draw even without the help of the king if it manages to get in front of it.

1...f2 2.h3+ g3 3.g1 h2 4.e2=

The story with the h-pawn is already different as the knight is trapped in the corner. If the pawn has reached the 2nd rank, the only way to hold the draw is if White can squeeze the king to the corner. In this example, White can achieve this by

1.c5! f3 2.d4 g2 3.e3 xh1 4.f2=

Probably the most important position from a practical point of view is the following one. If the pawn does not reach the 2nd rank, the knight holds the draw even without the help of the king.

1.a7 g2 2.g4 g3 3.e3 f3

3...f2 4.g4+=; 3...h2 4.f1+=

4.f1 f2
4...\textit{g}2 5.\textit{e}3+=

5.\textit{h}2=

The pawn cannot be promoted.

In the next couple of examples, the knight manages to stop a seemingly unstoppable pawn. If the black king were on f4, White would be winning. Let’s see why...

1.\textit{h}6 \textit{d}6 2.\textit{h}7 \textit{f}7+ 3.\textit{e}7 \textit{h}8 4.\textit{f}6!

Thanks to the e4 king, Black is able to squeeze the other king to h8 after

4...\textit{d}5! 5.\textit{g}7 \textit{e}6 6.\textit{x}h8 \textit{f}7=

In the following study by Prokop, the knight stands on the most distant part of the board from the pawn — but surprisingly he still manages to catch it.

1.\textit{g}6! a4 2.\textit{f}4 a3

If Black tries 2...\textit{c}2 with the idea of controlling the d3 square, the knight takes another direction 3.\textit{d}5 a3 4.\textit{b}4+ \textit{b}3 (4...\textit{c}3 5.\textit{a}2=) 5.\textit{d}3=

3.\textit{d}3!=

The knight has arrived just in time and it holds the a-pawn from the d3-c1-a2-b4 squares.
Black is threatening $\text{c5}$, chasing the knight away from the a3 and c3 squares from where it could stop the pawn — and when it moves to c7, … b3 is already unstoppable. However White can still hold the draw by a miraculous maneuver.

1. $\text{c7+}!$

Other moves are losing, e.g. 1. $\text{f7?}$ $\text{c5}$ 2. $\text{c7} \text{ b3} –+$

1... $\text{c4}$

1... $\text{d4}$ makes no sense because Black is not threatening …b3 because of the knight retreat to b5-a3. 2. $\text{f7} \text{ b3}$ 3. $\text{b5} =$

1... $\text{d6}$ 2. $\text{e8+!} \text{ e5}$ (2... $\text{e7}$ 3. $\text{c7}$ $\text{b3}$ 4. $\text{d5+} \text{ d6}$ 5. $\text{c3} =$; 2... $\text{c6}$ 3. $\text{f6} \text{ b3}$ 4. $\text{e4} \text{ b2}$ 5. $\text{d2} =$ just in time.) 3. $\text{c7} \text{ b3}$ 4. $\text{b5} \text{ b2}$ 5. $\text{c3} \text{ d4}$ 6. $\text{b1} =$

2. $\text{e8}!!$

Brilliant manoeuvre. Seemingly the completely wrong direction, but this is the only way to stop the b-pawn.

2... $\text{b3}$

The alternative is 2... $\text{c5}$ taking the d6 square under control, but now the knight goes 3. $\text{f6!} \text{ d4}$

And now again 4. $\text{e8}!! \text{ b3}$ 5. $\text{d6} =$ and the knight stops the pawn either from b5-a3 or from e4-d2.

3. $\text{d6+} \text{ b4}$

3... $\text{d3}$ 4. $\text{b5} \text{ b2}$ 5. $\text{a3} =$

4. $\text{e4} \text{ b2}$ 5. $\text{d2} =$

Never underestimate a knight!
In closing this chapter, here is my favorite motif in this type of position. The knight has just arrived to d5 and it looks like Black holds the draw. However White can promote his pawn by playing the fantastic

1. g8!!

The most unnatural move — going in front of the pawn — is the only way to win!

1.g8 f6+;

1.g6 allows 1...e7+! 2.f7 (2.f6 g8+=) 2...f5! 3.g8 h6+=

1...e7+

Otherwise f7 and the pawn promotes.

2.f8!

Avoiding the trap of 2.f7 f5! 3.g8 h6+. There are no more tricks left and a new queen will appear.

1.2. KNIGHT VS MULTIPLE PAWNS

There are no theoretical positions for this topic, but it is interesting to analyze some positions to become familiar with new ideas. From a practical point of view, the greatest importance attaches to the final two examples, which might easily arise in a game.

As we have already seen, a single knight can deliver checkmate if the opponent’s king is squeezed to the corner in front of an h-pawn. Some-
times the appearance of another pawn on the board does not change matters.

1. ♖f6! ♖h1 2. ♖g4!

Forcing the pawns to move.

2...f3

2...h2 is faster, 3. ♖f1+ and ♖f2 next.

3. ♖f1! f2 4. ♖xf2+ ♖h2 5. ♖e4!

The king must be kept in the cage.

5...♖h1 6. ♖f2 ♖h2

6...h2 7. ♖g3#

7. ♖d2

The knight goes to f1.

7...♖h1 8. ♖f1 h2 9. ♖g3#

Normally such situations are quite hopeless. The knight should not be able to hold two connected pawns which are supported by the king without the help of his own king. However, in this beautiful study by Chekhover, White still manages as if by miracle.

1. ♖e6! g4 2. ♖g7! f4

2...g3 3. ♖xf5 g2 4. ♖e3+=
3. \( \text{h5} \) \( f3 \) 4. \( f6 \) ! \( g3 \)

4...\( f2 \) 5. \( xg4 \) \( f1 \) \( \text{h6} \) 6. \( e3 \) +=

5. \( e4 \) \( g2 \)

5...\( f2 \) 6. \( xg3 \) =

6. \( d2 \) + \( d3 \) 7. \( xf3 \) \( e3 \) 8. \( g1 \) =

Just amazing! The c4 king was always standing in the ‘motif’.

The knight is a short range piece and therefore sometimes very unskillful. White promotes his pawns now without difficulties.

1. \( f7 \) \( g7 \) 2. \( e8! \)

Not 2. \( e7 \) because of 2...\( d5 \)+

In this study of Gurgenidze’s, White survives ‘miraculously’ thanks to an amazing knight maneuver. Actually Black’s plan is not to trap the knight with ...\( f7-g7 \), because then White is in time to eliminate the b5 pawn followed by rushing back to the corner. The main threat to deal with is the plan of ...\( e6-f5 \) followed by pushing the h-pawn, which can be only stopped by the knight. But how?!

1. \( a3!! \)

The only move. White must keep the b3 square free for the knight. On 1. \( b2 \) White is not in time to get back to the corner and Black can go
for the knight. 1...\(\text{f7}\)! 2.\(\text{c3}\) \(\text{g7}\) 3.\(\text{b4}\) \(\text{xh7}\) 4.\(\text{xb5}\) h5++; 2.

1...\(\text{e6}\)!

The critical test for White. 1...\(\text{f7}\) 2.\(\text{b4}\) \(\text{g7}\) 3.\(\text{xb5}\) \(\text{xh7}\) 4.\(\text{c4}\)= with an easy draw.

2.\(\text{f8+}\)!

2.\(\text{b4}\)? \(\text{f5}\) 3.\(\text{xb5}\) h5++; The knight has no chance of catching the pawn.

2...\(\text{f5}\) 3.\(\text{d7}\)

Only this long trip around the board helps.

3...h5 4.\(\text{c5}\) h4

This is the point and the reason why the b3 square had to be left free at the very beginning. 5.\(\text{d3}\)? h3 6.\(\text{f2}\) h2 7.\(\text{b4}\) \(\text{f4--}\)

5...h3 6.\(\text{d2}\) h2

6...\(\text{f4}\) 7.\(\text{f1}\)= As we know, the knight holds the draw without the help of the king if it gets to h2.

7.\(\text{f1}\)! h1\(\text{w}\) 8.\(\text{g3}\)++

The glorious finish of the big circle!

5

Practical position

Three pawns are running and White has also problems with saving his knight. However, as we know by now, a knight can cause surprises.

1.\(\text{f7}\)! e3 2.\(\text{f6+}\) \(\text{h8}\) 3.\(\text{h5!!}\)
A very interesting practical endgame. After all the analyses, it becomes clear that Black can hold the draw by extremely precise defense. In a practical game, however, White’s chances for the win are clearly higher than Black’s are to hold. This is simply because it is too easy to go wrong — and one mistake is decisive.
(8...\(\text{h3}\) 9.\(\text{h7}\) \(\text{g7}\) 10.\(\text{f6+}\) \(\text{xh7}\) 11.\(\text{f7}\) \(\text{g7}\) 12.\(\text{e7+-}\) ) 9.\(\text{d7}\)! and the king goes around to support the h-pawn. 9...\(\text{d3}\) (9...\(\text{f7}\) 10.\(\text{g5+-}\) ) 10.\(\text{e8)! The king arrived in time to control the g7 square. 11...\(\text{xg4}\) 12.\(\text{h7+-}\) ) 2.\(\text{g4}\) (2.\(\text{f6}\) \(\text{h5}\) White cannot make progress.) 2...\(\text{g6}\)! Another only move! As was mentioned before, Black should try to create holes between the pawns and occupy them with the king! 3.\(\text{h5}\) \(\text{h4+}\) 4.\(\text{f4}\) \(\text{g2+}\) 5.\(\text{g3}\) (5.\(\text{e4}\) \(\text{g5=}\) Black reaches the ideal blockading position.) 5...\(\text{e3}\) 6.\(\text{f4}\)

6...\(\text{d5}\)! This is a very important defensive method! Black must prevent the \(\text{h4}, \text{g5}\) plan... 7.\(\text{f3}\) \(\text{f6}\)! and using the fact that the pawn endgame is drawn, Black ties the king to his pawns. This motif also works well if we move all the pieces one rank further up. 8.\(\text{g5+}\) \(\text{h5}\) 9.\(\text{xf6}\) \(\text{g6}\) 10.\(\text{e4}\) \(\text{xf6=}\)

1...\(\text{e6+}\)!

This and the following two moves are the only way to survive!

2.\(\text{xf5}\) \(\text{d4+}\)!

The knight must disturb White’s plan of pushing his pawns.

3.\(\text{e4}\)

3.\(\text{e5}\) \(\text{f3+}\) 4.\(\text{e4}\) \(\text{h2}\) This is a nice achievement for Black. This way he gets enough time to rush with the king to h5. 5.\(\text{f5}\) \(\text{h6}\) (But not 5...\(\text{f6}\)? because of 6.\(\text{f4+}\) and the pawns are rolling.) 6.\(\text{f4}\) \(\text{h5}\) The draw seems to be close, but White can still create some problems... 7.\(\text{e5!}\) \(\text{f3+}\) 8.\(\text{d5}\) The king must control the d4 square. (8.\(\text{e6}\) \(\text{d4+=}\)
8...\( \text{h6} \) and the White king has strayed too far from the h-pawn, so the Black king must also turn back to stop the f-pawn. I know that this is very difficult! 9.\( \text{g}e4 \) \( \text{f}2 \) 10.\( \text{f}f4 \) \( \text{h}5= \) We already saw this.) 9...\( \text{g}7! \) (After the tempting 9...\( \text{d}4+? \) White gets his winning position again. 10.\( \text{e}5! \) \( \text{f}3+ \) 11.\( \text{f}4+ \) and g4 next.) 10.\( \text{f}6+ \) \( \text{f}8 \) and White cannot improve his position. 11.\( \text{h}5 \) (11.\( \text{f}5 \) \( \text{f}7= \)) 11...\( \text{d}4+ \) 12.\( \text{e}5 \) \( \text{c}6+ \) 13.\( \text{f}5 \) \( \text{f}7 \) 14.\( g4 \) \( \text{d}4+ \) 15.\( \text{e}5 \) \( \text{c}6+= \)

\[ \text{5...\( \text{c}4+ \) 6.\( \text{d}3 \)} \]

White should try to keep the knight away from bothering his h-pawn. 6.\( \text{d}4 \) \( \text{d}2 \) 7.\( \text{h}5 \) \( \text{h}6= \) The same as before.

\[ \text{6...\( \text{d}6 \) 7.\( \text{d}4 \)} \]

Everything seems to be going well for White. He is progressing with the pawns step-by-step without creating a hole, but Black has an amazing and difficult only move again... 7.\( \text{h}5 \) \( \text{h}6= \)

\[ \text{7...\( \text{f}6! \) 8.\( \text{h}5 \)} \]

Trying to make use of the fact that Black has no...\( \text{h}6. \) 8.\( \text{d}5 \) \( \text{b}5 \) 9.\( \text{h}5 \) \( \text{c}3+ \) 10.\( \text{d}4 \) (10.\( \text{d}6 \) White cannot l the pawns alone... 10...\( \text{e}2= \)) 10...\( \text{b}5+ \) transposes to the main line.

\[ \text{8...\( \text{b}5+ \)} \]

‘transposes to the main line.’

\[ \text{9.\( \text{e}3 \) \( \text{d}6 \) 10.\( \text{f}3 \) \( \text{c}4 \)} \]
All the Black moves have a goal. The knight now heads for the e3 square...

11.h6

11.\(\text{g}3\) e3 This is the ideal position for the knight. It attacks one of the pawns and prepares for \(\text{g}2\) check after \(\ldots\text{h}4\).

11...\(\text{g}6\) 12.g5 \(\text{d}6\) 13.\(\text{g}4\)

and again, it seems to be over, but...

13...\(\text{e}4\)!

Of course not 13...\(\text{f}5\)? 14.h7+-

14.f5+

14...\(\text{h}7\)

and just like in the 12. \(\text{g}3\) e3 line, Black has managed to attain his ideal setup. The king is ahead of the h-pawn, while the knight attacks the g-pawn and prevents the \(\ldots\text{h}5\) move by \(\text{g}3\).

15.\(\text{f}4\)

15.\(\text{h}5\) \(\text{g}3+\)=

15...\(\text{d}6\) 16.\(\text{e}5\)

The last attempt to make progress.

16...\(\text{f}7+\) 17.\(\text{f}6\) \(\text{d}6\)

and White must settle for a draw.

18.\(\text{e}6\)

18.g6+ \(\text{h}6\) 19.g7 \(\text{e}8+=\)

18...\(\text{e}4\) 19.g6+ \(\text{x}h6\)=

7

Practical position

Similar to the previous example, but White achieves his ideal position.

1.c5+ \(\text{c}7\) 2.b5 \(\text{a}5\)